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DISPLACEMENT ANALYSIS OF THE HUMAN KNEE JOINT BASED ON THE SPATIAL KINEMATIC MODEL BY USING VECTOR METHOD

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Abstract: Kinematic model of the human knee joint, considered as one-degree-of-freedom spatial parallel mechanism, is used to analyse the spatial displacement of the femur with respect to the tibia. The articular surfaces of femoral and tibia condyles are modelled, based on selected references, as spherical and planar surfaces. The condyles are contacted in two points and are guided by three ligaments modelled as binary links with constant lengths. In particular, the mechanism position problem is solved by using the vector method. The obtained kinematic characteristics are adequate to the experimental results presented in the literature. Additionally, the screw displacements of relative motion in the knee joint model are determined.

Key words: Human Knee Joint Model, Kinematic Analysis, Parallel Mechanism, Constraint Equations, Vector Method

1. INTRODUCTION

The human knee joint (Fig. 1) provides a large relative movement of two bones (femur and tibia) that are constrained to remain in contact at two points and guided with three ligaments (ACL - anterior cruciate ligament, MCL – medial collateral ligament and PCL – posterior cruciate ligament), as mentioned in Sancisi, Parenti-Castelli (2010), Parenti-Castelli and Di Gregorio (2000) and Góra (2008). The knee is important in daily living activities and because of the high incidence of injuries and diseases involving this joint, which considerably affect locomotion. Restoration of normal knee joint function and range of motion, as pursued by reconstructive surgery and rehabilitation, can be achieved by re-establishing the natural relationship between the geometrical shape of the articular surfaces and the geometry of the ligaments, as presented in Woo et al. (2006). Kinematic models of the knee joint are very useful for defining diagnostic procedures, for pre-surgical planning, for functional assessment after knee surgery, and for designing prosthetic replacement devices.

Kinematic models presented as equivalent mechanisms (M1 – planar, M2 – spherical or M3 - spatial), with one degree of freedom (1-dof), can be used to analyse the relative motion of the femur with respect to the tibia, give in Parenti-Castelli and Di Gregorio (2000) and Sancis and Parenti-Castelli (2010). The considered mechanism contains two nonsymmetrical platforms (the femur and the tibia) with two contact points and four legs (ACL, PCL, MCL and PF- patella-femoral joint).The passive motion of the tibia-femoral joint (TF) is not constrained from that of patella-femoral chain if knee flexion is externally imposed. Thus the two sub-joints (TF an PF) of the knee can be analysed separately and in particular, tibio-femoral joint (with one degree-of-freedom) can be used to replicate the passive motion without taking patella-femoral (PF) joint into consideration. Since no loads are applied to the joint during passive motion, the muscles remain inactive, they do not guide the knee and, as a consequence, they are not considered in this study.

It has been observed that three ligaments (ACL, PCL and MCL) can be considered as isometric fibres (or cables) during the flexion of the unloaded knee. Thus, three ligaments are modelled as binary links, each connected to the tibia and femur by a spherical joint. The other bundles are not tight and reach the limit between laxity and tension at the most. As a consequence, only isometric bundles guide the passive motion of the knee, while the others can be ignored in the model.

More recent studies concern a knee joint modelling taking into account elasticity in ligaments, like in Sancisi and Parenti-Castelli (2011) or in Saldias et al. (2013).

Synthesis task, where for given functional characteristics wanted are selected dimensions of knee joint model, seems to be the most challenging. Innovative approaches are presented in Parenti-Castelli and Sancisi (2013) or in Saldias et al (2014).

The present paper aims to enhance the knowledge of knee joint mobility by equivalent 1-dof spatial mechanism, based on the knee model proposed in Parenti-Castelli and Di Gregorio (2000) and Sancisi (2013) and applied in Di Gregorio and Parenti-Castelli (2003). The surfaces of the femur and tibia condyles are modelled by rigid spherical and planar in point contact with one another. In particular, the scope was to use vector method, given in Morecki et al. (2002), to analyse position and displacement of the femur with respect to the tibia, and the path of the instantaneous screw axis.

2. FORMULATION OF KINEMATIC MODEL

Kinematic model (M3) of knee joint based on the measurement results from Parenti-Castelli and Di Gregorio (2000) was

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used to analyse the relative position and displacement of the femur with respect to the tibia. There are two frames (Fig. 1) base reference system $\{xyz\}$ embedded in the tibia and reference system $\{x^b y^b z^b\}$ fixed to the femur.

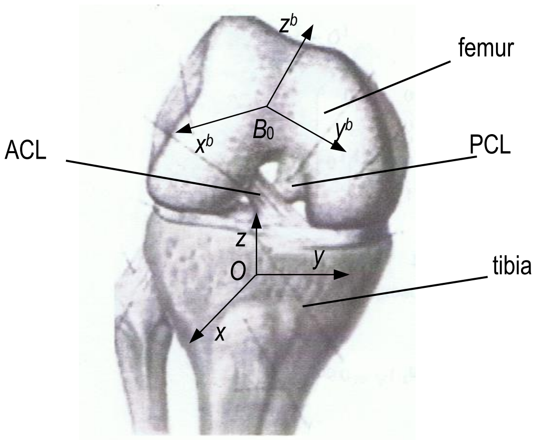

Fig. 1. Schematic anterior view of the knee in flexion (Woo et al., 2006)

The points A_i (i = 1, 2, 3) denote the centres of the joints, modelling the ligament insertions into the tibia; $\mathbf{a}_i$ – the position vector of point A_i with respect to the origin of the system$\{xyz\}$. The points B_i (i = 1, 2, 3) denote the centres of joints, modelling the ligament insertions into the femur and $\mathbf{b}_i^b$ – the position vector of the point B_i with respect to the origin of the system $\{x^b y^b z^b\}$, $\mathbf{b}_i$ – the position vector with respect to the origin of the base system $\{xyz\}$. The respective relation is described by using the formula:

$$\mathbf{b}_i = \mathbf{b}_0 + \mathbf{R}^b \mathbf{b}_i^b, \qquad i = 1, 2, 3 \tag{1}$$

where $\mathbf{b}_0$ – position vector of point B_0 assumed as the origin of coordinate system $\{x^b y^b z^b\}$, $\mathbf{R}^b$ – orientation matrix of the system $\{x^b y^b z^b\}$ with respect to the system $\{xyz\}$.

The position and displacement analysis of the spatial mechanism can be accomplished in the following way. The sphere surfaces are removed from contact points with the planes π_j (j = 4, 5) and the sphere centres B_j (j = 4, 5) are treated as coupler points of the transformed mechanism. The position of this mechanism (now with three degree of freedom) is described by three angles ϕ_i (i = 1, 2, 3), shown in Fig. 2, which can be treated as additional independent variables, with values to be find from closure equations. The position vectors $\mathbf{b}_4$ and $\mathbf{b}_5$ of the platform points B_4 and B_5 can be found using the vector method described below, and their respective distances from the planar surfaces (π_4, π_5) can be described as functions of ϕ_i giving the closure equations of the mechanism, as described below.

Since the sphere slides on the plane π_j (j = 4, 5) its centre point B_j (j = 4, 5) always belongs to a parallel plane and is located at a distance equal to the radius r_j. These conditions can be written, as in Sancisi, Parenti-Castelli (2010) [10] and Parenti-Castelli, Di Gregorio (2000) [5] and Góra (2008) [2], as follows:

$$F_i(\phi_1, \phi_2, \phi_3) = ||\mathbf{b}_j - \mathbf{o}_j|| - r_j = 0, \quad j = 4, 5 \tag{2}$$

$$F_j(\phi_1, \phi_2, \phi_3) = n_{jx}(b_{jx} - x_j) + n_{jy}(b_{jy} - y_j) + n_{jz}(b_{jz} - z_j) = 0 \qquad j = 4, 5 \tag{3}$$

where: $\hat{\mathbf{n}}_j = [n_{jx}, n_{jy}, n_{jz}]^{\mathrm{T}}$, $\mathbf{b}_j = [b_{jx}, b_{jy}, b_{jz}]^{\mathrm{T}}$, $\mathbf{o}_j = [x_j, y_j, z_j]^{\mathrm{T}}$,

$\mathbf{b}_j$ – position vector of point B_j (j = 4, 5), i.e. curvature centre of femur condyle surface described in the system $\{xyz\}$;

$\hat{\mathbf{n}}_j$ – unit vector as the normal to the plane π_j,

$\mathbf{o}_j$ – position vector of the plane point O_j ($O_4 \in \pi_4$, $O_5 \in \pi_5$), described in the system $\{xyz\}$ (tibia).

Additional angles ϕ_1, ϕ_2, ϕ_3 (Fig. 2) are defined respectively as the angles between the pairs of unit vectors: $(\hat{\mathbf{a}}_{21}, \hat{\mathbf{d}}_2)$, $(\hat{\mathbf{a}}_{23}, \hat{\mathbf{d}}_2)$ and $(-\hat{\mathbf{a}}_{21}, \hat{\mathbf{d}}_1)$, where:

$$\begin{aligned} &\mathbf{d}_1 = \mathbf{b}_1 - \mathbf{a}_1, \ \ \mathbf{d}_2 = \mathbf{b}_2 - \mathbf{a}_2, \\ &\mathbf{a}_{21} = \mathbf{a}_1 - \mathbf{a}_2, \ \ \mathbf{a}_{23} = \mathbf{a}_3 - \mathbf{a}_2, \\ &\mathbf{b}_{21} = \mathbf{b}_1 - \mathbf{b}_2, \ \ \mathbf{b}_{23} = \mathbf{b}_3 - \mathbf{b}_2, \\ &\mathbf{d}_{12} = \mathbf{b}_2 - \mathbf{a}_1, \ \ \mathbf{d}_{21} = \mathbf{b}_1 - \mathbf{a}_2. \end{aligned} \tag{4}$$

The general formula for finding one of three unit vectors can be treated as a subroutine used to calculate unknown unit vector $\hat{\mathbf{w}}$, when two unit vectors ($\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$) and two dot products of each these vectors with the unknown unit vector $\hat{\mathbf{w}}$ ($\hat{\mathbf{u}} \cdot \hat{\mathbf{w}}, \hat{\mathbf{v}} \cdot \hat{\mathbf{w}}$) are known. The unknown unit vector $\hat{\mathbf{w}}$ is determined by formula, given in Morecki et al. (2002):

$$\hat{\mathbf{w}} = [((\hat{\mathbf{u}} \cdot \hat{\mathbf{w}}) - (\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})(\hat{\mathbf{v}} \cdot \hat{\mathbf{w}}))\hat{\mathbf{u}} + ((\hat{\mathbf{v}} \cdot \hat{\mathbf{w}}) - (\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})(\hat{\mathbf{u}} \cdot \hat{\mathbf{w}}))\hat{\mathbf{v}} \pm \pm (\hat{\mathbf{u}} \times \hat{\mathbf{v}})\sqrt{D}\,][((1 - (\hat{\mathbf{u}} \cdot \hat{\mathbf{v}}))(1 + (\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})))^{-1}$$

where

$$D = 1 - (\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})^2 - (\hat{\mathbf{u}} \cdot \hat{\mathbf{w}})^2 - (\hat{\mathbf{v}} \cdot \hat{\mathbf{w}})^2 + 2(\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})(\hat{\mathbf{u}} \cdot \hat{\mathbf{w}})(\hat{\mathbf{v}} \cdot \hat{\mathbf{w}})$$

By using this formula the following unit vectors can be determined in the specified order: $\hat{\mathbf{d}}_2, \hat{\mathbf{d}}_1, \hat{\mathbf{b}}_{23}, \hat{\mathbf{b}}_{24}, \hat{\mathbf{b}}_{25}, \hat{\mathbf{b}}_0$.

The position vectors $\mathbf{b}_m$ (m = 0, 1,...5) of points B_m are described in the base system $\{xyz\}$. The solution procedure for determining the position vectors $\mathbf{b}_m$ of the femur points in the base system is presented in Tab. 1.

Tab. 1. The following steps of the solution procedure for the direct position problem by using vector method

Step	$\hat{\mathbf{u}}$	$\hat{\mathbf{v}}$	$\hat{\mathbf{w}}$	$\mathbf{b}_m$
1	$\hat{\mathbf{a}}_{21}$	$\hat{\mathbf{a}}_{23}$	$\hat{\mathbf{d}}_2$	$\mathbf{b}_2(\phi_1, \phi_2) = \mathbf{a}_2 + d_2\hat{\mathbf{d}}_2$
2	$\hat{\mathbf{a}}_{12}$	$\hat{\mathbf{d}}_{12}$	$\hat{\mathbf{d}}_1$	$\mathbf{b}_1 = \mathbf{a}_1 + d_1\hat{\mathbf{d}}_1$
3	$\hat{\mathbf{b}}_{21}$	$-\hat{\mathbf{d}}_{32}$	$\hat{\mathbf{b}}_{23}$	$\mathbf{b}_3 = \mathbf{b}_2 + b_{23}\hat{\mathbf{b}}_{23}$
4	$\hat{\mathbf{b}}_{21}$	$\hat{\mathbf{b}}_{23}$	$\hat{\mathbf{b}}_{24}$	$\mathbf{b}_4 = \mathbf{b}_2 + b_{24}\hat{\mathbf{b}}_{24}$
5	$\hat{\mathbf{b}}_{21}$	$\hat{\mathbf{b}}_{23}$	$\hat{\mathbf{b}}_{25}$	$\mathbf{b}_5 = \mathbf{b}_2 + b_{25}\hat{\mathbf{b}}_{25}$
6	$\hat{\mathbf{b}}_{23}$	$\hat{\mathbf{b}}_{24}$	$\hat{\mathbf{b}}_0$	$\mathbf{b}_0 = \mathbf{b}_2 + b_0\hat{\mathbf{b}}_0$

The analysed range of the permissible displacements was divided into a finite number of discrete positions. The system of nonlinear equations (2) may be solved for two unknowns ϕ_2 and ϕ_3 assuming the selected value of ϕ_1.

On the basis of the algorithm described above a computer program in MATLAB was written. The solutions satisfied the geometrical conditions are used to determine the successive positions of the considered mechanism and the respective femur displacements as the function of the knee joint flexion angle. This algorithm can be also used for the parameter estimation procedure of the equivalent mechanism, for example to determine the coordinates of the ligament insertion points, that satisfied the correct mobility of the joint knee.

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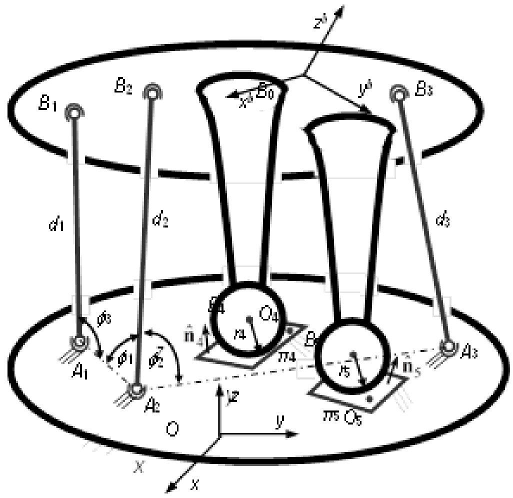

Fig. 2. Model of the knee joint. Notations: d_i (i=1, 2, 3) - isometric ligament modelled as binary link, with A_i – joint at the tibia, B_i – joint at the femur. The surfaces of the femur condyles are modelled by spherical (r_4 , r_5) surfaces and tibia condyles as planar (π_4 , π_5) surfaces in contact with one another. The coordinate system {*xyz*} is fixed to the tibia and the system {$x^b y^b z^b$} is fixed to femur (Di Gregorio and Parenti-Castelli, 2003)

Position of the femur and its relative displacement with respect to the tibia, according to the model of the knee joint, can be described by using one input variable, for example flexion angle (α), like in Di Gregorio and Parenti-Castelli (2003). The solution for the position problem in the system {*xyz*} can be described by

- position vectors ($\boldsymbol{b}_m$, m = 0, 1,...5) of the femur points;
- position vector ($\mathbf{o}^b$) of the origin of the femur system;
- orientation matrix ($\mathbf{R}^b$) of the femur system with respect to the tibia system.

The femur orientation with respect to the tibia can be described by a sequence of three rotation angles: α – the flexion of the knee as the rotation angle around the *y* axis of the system {*xyz*}, β – rotation angle around the *x* axis, and γ – rotation angle around the *z* axis. In accordance with Parenti-Castelli and Di Gregorio (2000), the following yields:

$$\mathbf{R}^b = \mathbf{R}_z(-\gamma)\,\mathbf{R}_x(\beta)\,\mathbf{R}_y(\alpha) \tag{5}$$

Moreover these three angles are assumed equal to zero in the full extension configuration. The considered orientation matrix, defined in Di Gregorio and Parenti-Castelli (2003) and Parenti-Castelli and Di Gregorio (2000), has the following expression:

$$\mathbf{R}^b = \begin{bmatrix} c\alpha\, c\gamma + s\alpha\, s\beta\, s\gamma & c\beta\, s\gamma & c\gamma\, s\alpha - c\alpha\, s\beta\, s\gamma \\ c\gamma\, s\alpha\, s\beta - c\alpha\, s\gamma & c\beta\, c\gamma & -c\alpha\, c\gamma\, s\beta - s\gamma\, s\alpha \\ -c\beta\, s\alpha & s\beta & c\alpha\, c\beta \end{bmatrix} \tag{6}$$

where the following notation is used: $c = \cos$, $s = \sin$.

If the elements of the matrix (6) are known:

$$\mathbf{R}^b = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \tag{7}$$

then the orientation angles can be calculated as follow:

$$\beta = \{\arcsin(m_z),\ \pi - \arcsin(m_z)\}$$

$$\alpha = \{\arcsin(-l_z/\cos\beta),\ \pi - \arcsin(-l_z/\cos\beta)\} \tag{8}$$

$$\gamma = \{\arcsin(m_x/\cos\beta),\ \pi - \arcsin(m_x/\cos\beta)\}$$

Equating the respective elements of the matrices (6) and (7) the values of the orientation angles (8) are calculated.

3. NUMERICAL EXAMPLES

The point coordinates and the link lengths of the knee joint model (Fig. 2), assumed as data according to Parenti-Castelli and Di Gregorio(2000), are given in Tab. 2 and 3.

Tab. 2. Coordinates of the vectors $\mathbf{a}_i$ described in the system {*xyz*}, $\mathbf{b}_j^b$ – in the system {$x^b y^b z^b$}, the lengths d_i and r_j [mm]

i, *j*	$\mathbf{a}_i$	$\mathbf{b}_j^b$	d_i	r_j
1	$[-3\ \ 0\ \ 0]^T$	$[19.2;16.9;\ 26.8]^T$	38.8	-
2	$[20.2\ \ 12.2\ \ -18.3]^T$	$[16.8;\ -6.5;\ 10.8]^T$	34.8	-
3	$[-11.4\ \ -2.4\ \ -53.6]^T$	$[8.2;\ -34.1;\ 13.8]^T$	77,0	-
4	-	$[5.1;\ -15.6;\ 19.1]^T$	-	24.6
5	-	$[3.0;\ 35.5;\ 27.1]^T$	-	30.3

Tab. 3. Coordinates of the vectors $\boldsymbol{o}_j$ and the unit vectors $\hat{\mathbf{n}}_j$ of the planes π_4, π_5 described in the system {*xyz*}

j	$\mathbf{o}_j$ [mm]	$\hat{\mathbf{n}}_j$ [-]
4	$[5.1\ -15.6\ 19.1]^T$	$[0.10\ \ -0.25\ \ 0.96]^T$
5	$[3.0\ \ 35.5\ 27.1]^T$	$[0.21\ \ \ 0.16\ \ \ 0.97]^T$

The position vectors $\boldsymbol{b}_m$ of the femur points calculated by using the algorithm presented in Table 1 are given in Tab. 4.

Tab. 4. Coordinates of the position vectors $\boldsymbol{b}_m$ of the femur points calculated for the determined values of additional angles

Additional angles	*M*	$\boldsymbol{b}_m$ [mm]
$\phi_1 = 45^o$	1	$[34.7;\ \ 8.2;\ \ 7.5]^T$
$\phi_2 = 101^o$	2	$[19.1;-15.1;\ \ 3.3]^T$
$\phi_3 = 51^o$	3	$[22.1;\ -43.6;\ \ 1.8]^T$
	4	$[28.9;\ -27.1;\ 11.2]^T$
	5	$[36.7;\ \ 20.9;\ 28.6]^T$
	0	$[\ 9.1;\ \ -14.3;\ 21.7]^T$

Orientation matrix of the femur system with respect to the base system, calculated by using the femur point coordinates is given by formula (7). The respective knee joint angles, calculated by using formula (8), are: $\alpha = 18^o$, $\beta = 1.2^o$, $\gamma = 2.5^o$.

The femur pose with respect to the tibia is described by using the position vector $\mathbf{o}_k^b$ (k – number of poses) and the orientation matrix $\mathbf{R}_k^b$ dependent on the flexion angle as independent variable α_k (k = 1, …, n).

The determined coordinates of the system {$x^b y^b z^b$} origin of the (femur) in relation to the flexion angle (α) are presented in Fig. 3. The obtained characteristics are compared to the simulation results from Parenti-Castelli and Di Gregorio (2000). The greatest displacement in *z* direction, reaching 17 mm, is adequate to the reference model curve from Parenti-Castelli and Di Gregorio (2000). The obtained characteristics of the knee displacements

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in x and y directions have the same profiles but are biased by ca 1 mm with respect to the reference study.

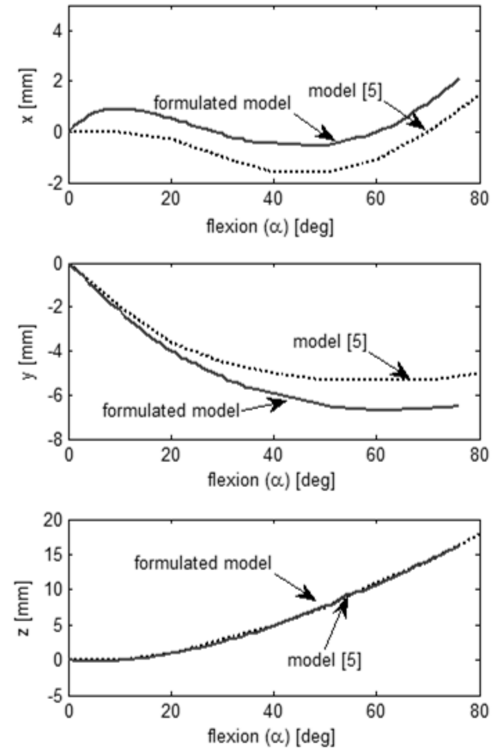

Fig. 3. The coordinates of the position vector $\boldsymbol{b}_0$ of the femur point B_0 in relation to the flexion angle α. Comparison of the own results with simulation from Parent-Castelli and Di Gregorio (2000)

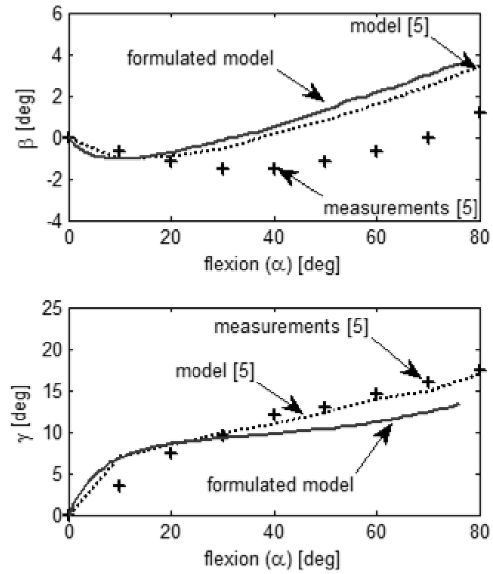

Fig. 4. Orientation angles (β and γ) of the femur with respect to the tibia in relation to the flexion angle (α). Comparison of the own results with simulation and meausurements from Parent-Castelli and Di Gregorio (2000)

The knee joint angles β and γ as functions of the flexion angle α are illustrated in Fig. 3. These characteristics, achieved by using the formulated knee model, are compared to the results from Parent-Castelli and Di Gregorio (2000) consititng of simulation and experimental results in Wilson et al. (1998). Generaly, the formulated model gives adequate results to the reference model. However, at flexion angles above 40° some deviation is noticeable, especially for γ angle. This can be a consequence of an error propagation in the utilized numerical approach.

4. SCREW DISPLACEMENTS FOR KNEE FLEXION

For a finite femur body displacement, the screw parameters can be determined by using the coordinates of three non-collinear points fixed to a body in some initial (n) and final (n+1) positions.

The screw axis of the finite displacement of the body between its two positions (with the upper left index ^{n} and $^{n+1}$) can be determined by using the formula, given in Morecki et al. (2002):

$$\hat{\mathbf{e}}_{n,n+1}\, tg\frac{\theta_{n,n+1}}{2} = \frac{({}^{n+1}\mathbf{b}_{jk} - {}^{n}\mathbf{b}_{jk}) \times ({}^{n+1}\mathbf{b}_{ji} - {}^{n}\mathbf{b}_{ji})}{({}^{n+1}\mathbf{b}_{jk} - {}^{n}\mathbf{b}_{jk}) \cdot ({}^{n+1}\mathbf{b}_{ji} + {}^{n}\mathbf{b}_{ji})} \tag{9}$$

where:
${}^{n}\mathbf{b}_{jk} = {}^{n}\mathbf{b}_{k} - {}^{n}\mathbf{b}_{j}$ – position vector of point B_j relative to B_k, corresponding to n –th position of the body;
$\hat{\mathbf{e}}_{n,n+1}$ – unit vector of the screw displacement axis;
$\theta_{n,n+1}$ – angular displacement of the body from position n to position n + 1 around this axis

Position vector of the axis point is described as

$$\mathbf{p}_{n,n+1} = \frac{1}{2}[{}^{n+1}\mathbf{b}_{i} + {}^{n}\mathbf{b}_{i} + \hat{\mathbf{e}}_{n,n+1} \times ({}^{n+1}\mathbf{b}_{i} - {}^{n}\mathbf{b}_{i})ctg\frac{\theta_{n,n+1}}{2} + - \hat{\mathbf{e}}_{n,n+1} \cdot ({}^{n+1}\mathbf{b}_{ji} + {}^{n}\mathbf{b}_{ji})\hat{\mathbf{e}}_{n,n+1}] \tag{10}$$

($\boldsymbol{p}_{n,n+1}$) - position vector of this axis with respect to the base system and
($u_{n,n+1}$) - body displacement along this screw axis (Fig. 5).

The value ($u_{n,n+1}$) of the linear displacement of the body along the screw axis is determined by the formula:

$$u_{n,n+1} = \hat{\mathbf{e}}_{i} \cdot ({}^{n+1}\mathbf{b}_{i} - {}^{n}\mathbf{b}_{i}) \tag{11}$$

Linear displacement of the body along the screw axis is determined by the formula

$$u_{n,n+1} = \hat{\mathbf{e}}_{n,n+1} \cdot ({}^{n+1}\mathbf{b}_{i} - {}^{n}\mathbf{b}_{i}) \tag{12}$$

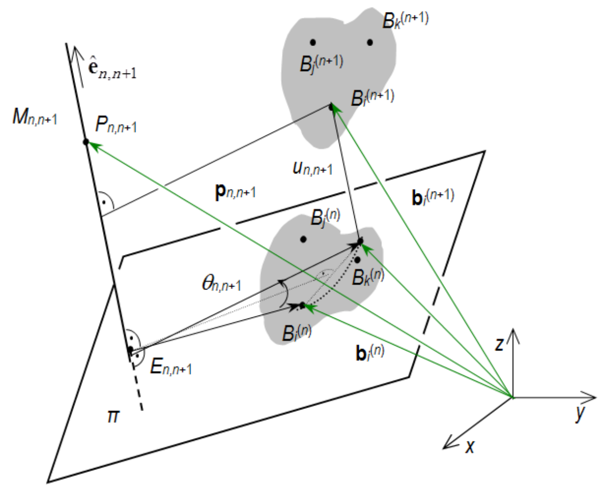

Fig. 5. Axis of the screw displacement of the body, described by using the unit vector ($\hat{\boldsymbol{e}}_{n,n+1}$) and the position vector ($\boldsymbol{p}_{n,n+1}$) of the axis with respect to the base system

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According to the described procedure, axes of the femur screw displacements are determined with respect to the base system. Selected screw parameters (**e**, **p**, *u*) are given in Tab. 5, where for example n=1 corresponds to a finite displacement of the flexion angle between α=25° and α=30°. The obtained screw pitches (u) have relatively small magnitudes, what corresponds to a pure rotation about the screw axis.

Tab. 5. Parameters of the femur screw displacement with respect to the base frame {*xyz*} determined for different flexion angles α_n

n	α_n [°]	$\mathbf{e}_{n,n+1}$ [-]	$\mathbf{p}_{n,n+1}$ [m]	$u_{n,n+1}$ [m]
1	25	$\begin{bmatrix} 0.2185 \\ 0.9740 \\ -0.0602 \end{bmatrix}$	$\begin{bmatrix} 0.8949 \\ -6.9560 \\ 4.9147 \end{bmatrix}$x10^{-3}	-0.6800 x10^{-3}
2	30			
8	60	$\begin{bmatrix} 0.2692 \\ 0.9585 \\ -0.0938 \end{bmatrix}$	$\begin{bmatrix} 9.1315 \\ -40.3760 \\ 7.6747 \end{bmatrix}$x10^{-3}	0.0279 x10^{-3}
9	65			
14	90	$\begin{bmatrix} 0.3536 \\ 0.9278 \\ -0.1190 \end{bmatrix}$	$\begin{bmatrix} 12.9343 \\ -38.6413 \\ 9.9044 \end{bmatrix}$x10^{-3}	0.5702 x10^{-3}
15	95			

The following graphical representation of the screw axes enables better understating of a spatial character of this joint motion.

The femur screw displacements with respect to the tibia reference system are illustrated in Fig. 6 by the screw axes with direction unit vectors ($\mathbf{e}_a$, $\mathbf{e}_b$, $\mathbf{e}_c$) and position vectors (P_a, P_b, P_c) for the three finite displacements (α = 25^0 and 30^0; α = 60^0 and 65^0; α = 90^0 and 95^0). It can be noticed, that the screw axes are mainly directed along lateral (y) axis of the base reference frame. Simultaneously, the screw axis position changes slightly for each knee flexion, what corresponds to a position change of an instantaneous rotation point in the knee joint. Additionally, the screw axes are positioned inside the joint, it means between the three ligaments.

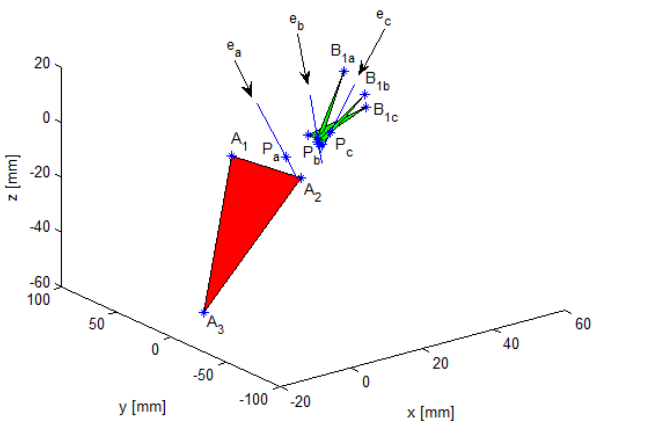

Fig. 6. Axes (e_a, e_b, e_c) of the femur screw displacements with respect to the base frame. Notations: *a)* α = 25^0 and 30^0; *b)* α = 60^0 and 65^0; *c)* α = 90^0 and 95^0

For further explanation of the knee joint model displacement (Fig. 2), the linear displacements of the curvature centres B_4 and B_5 of the femur condyle surfaces are investigated. Their coordinates are presented in Fig. 7 in the tibia reference system {*xyz*} as functions of the flexion angle. The obtained changes in the coordinates are related to a quasi-rolling of the considered bones.

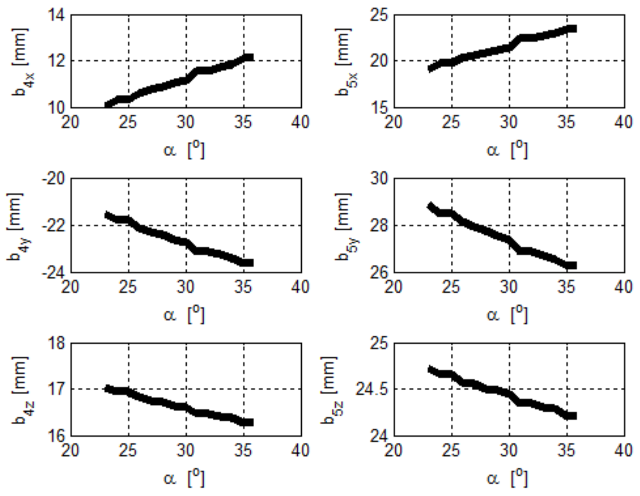

Fig. 7. Coordinates of point B_j (j=4,5), i.e. curvature centre of femur condyle surface, described in the system {*xyz*}as functions of the flexion angle α

5. CONCLUSIONS

Kinematic model of the human knee joint, considered as parallel mechanism, was formulated to determine the spatial displacement of the femur with respect to the tibia. The vector method was utilized for solving the direct position analysis (DPA) of the considered mechanism. The parameters of finite screw displacements are derived for better explanation of the knee joint spatial motion.

Numerical simulations proved effectiveness of the prepared algorithm. The elaborated algorithm can be used in the parameter estimation procedure of the equivalent mechanism, for example to determine the coordinates of the ligament insertion points, that satisfied the correct mobility of the joint knee. The formulated model enables to determine allowed ranges of the knee displacements and possible collision between the ligaments and the bones.

This algorithm can also be used for sensitivity analysis of the dimension tolerances on accuracy of the equivalent mechanism.

It seems useful to consider the linear displacement along the instantaneous screw axis of the joint motion, as it is allowed in the actual joint. Estimation of the model parameters can improve the results from the numerical analysis.

Further extensions of the kinematic model may led to solve static and elasto-static problems. The modified equivalent mechanism with femur and tibia condyles modelled as spherical or general shape surfaces may give better agreement with experiments.

REFERENCES

1. **Di Gregorio R., Parenti-Castelli V.** (2003), A spatial mechanism with higher pairs for modelling the human knee joint, *Trans. ASME Jnl of Biomechanical Eng.*, 125, 232-237.

DOI 10.1515/ama-2017-0050

2. **Góra M.** (2008), *Kinematic analysis of the multi-rod suspension mechanisms of the cars*, Doct. Diss, Cracow University of Technology.
3. **Morecki A., Knapczyk J., Kędzior K.** (2002), *Theory of mechanisms and manipulators*, WNT, Warsaw 2002.
4. **Ottoboni A., Parenti-Castelli V., Sancisi N.** (2010), Articular surface approximation in equivalent spatial parallel mechanism models of the human knee joint: an experiment-based assessment, *Proc. IMechE, Part H: Engineering in Medicine,* 224, 1121-1132.
5. **Parenti-Castelli V., Di Gregorio R.** (2000), Parallel mechanisms applied to the human knee passive motion simulation, *Advances in Robot Kinematics, Kluwer Academic Publ.* Dordrecht, 333-343.
6. **Parenti-Castelli V., Sancisi N.** (2013), Synthesis of spatial mechanisms to model human joints. In: McCarthy J. (eds) *21st Century Kinematics*, Springer, London.
7. **Saldias D., Martins D., de Mello Roesler C., da Silva Rosa F., Ocampo Moré A.,** (2013), Modeling of human knee joint in sagittal plane considering elastic behavior of cruciate ligaments, 22nd *International Congress of Mechanical Engineering*, November 3-7, 2013, Ribeirão Preto, SP, Brazil.
8. **Saldias D., Radavelli L., Roesler C., Martin D.** (2014), Kinematic synthesis of the passive human knee joint by differential evolution and quaternions algebra: a preliminary study, 5th IEEE RAS & EMBS *International Conference on Biomedical Robotics and Biomechatronics* (BioRob), 12-15 Aug. 2014, Brazil.
9. **Sancisi N., Parenti-Castelli V.** (2010), A 1-Dof parallel spherical wrist for the modelling of the knee passive motion, *Mechanism and Machine Theory*, 45, 658-665.
10. **Sancisi N., Parenti-Castelli V.,** (2011), A sequentially-defined stiffness model of the knee, *Mechanism and Machine Theory*, 46(12), 1920-1928.
11. **Wilson D.R., Feikes J.D., O'Connor J.J.** (1998), Ligaments and articular contact guide passive knee flexion, *Journal of Biomechanics*, 31, 1127-1136.
12. **Woo S., Abramowitch S., Kilger R., Liang R.,** (2006), Biomechanics of knee ligaments: injury, healing, and repair, *Journal of Biomechanics*, 39(1), 1–20.